Time-Domain Model Synthesis of Microstrip

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***Abstract*—The time-domain model of a microstrip line is synthesized by means of deconvolution of response voltages and currents simulated by finite-difference time-domain (FDTD) method. With the model, the response voltages and currents of any excitation of the line with any loads can be easily and rapidly simulated, instead of using time-consuming FDTD again. This model has been applied to simulate the responses of several excitations and the voltage response of a diode terminating the microstrip line. The results show good agreement with the direct FDTD simulation.**

I. Introduction

FOR A DESIGNER of high-speed very large scale integrated (VLSI) interconnects, the electromagnetic details of the system must be considered, because a large signal delay and radiation may appear as digital signal clock speeds are increased beyond 200 MHz. Of course, one-dimensional (1-D) circuit equations are not valid. We need to solve the Maxwell's equations to get the exact solution. One of the most suitable schemes for our purpose is the finite-difference time-domain (FDTD) method, which has been widely used to model various electromagnetic phenomena and interactions [1]–[3].

When the simulated structure is large and complex, however, FDTD is time consuming and occupies too much computer memory. If the source or loads change, the calculation must be carried out again, thus wasting time.

It is well known, in frequency-domain, that a complex linear network can be divided into several simple subnetworks. Once the matrix parameters of these subnetworks are found, the characteristics of the complex network can be obtained by matrix operation. Although we can get wide-band frequency response from the FDTD simulation by means of Fourier transform, frequency-domain methods are not as straightforward as time-domain methods, and transient simulation of active nonlinear circuits mixed with frequency-domain characterized interconnect components is awkward. Thus, it is better to characterize network functions in the time-domain in the first place [4].

In this letter, we introduce the time-domain model of a basic VLSI interconnect, the microstrip line. The model is composed of transient characteristic impedance and delay function. By means of the numerical deconvolution, we obtain the model from response voltages and currents simulated by FDTD. With this model, the response of any excitation of microstrip line with any loads can be easily and rapidly simulated by one-dimensional (1-D) convolution operation, instead of by three-dimensional (3-D) FDTD operation.

The model has been used to simulate the responses of two kinds of excitation and the response voltage of a diode terminating the line. The results show good agreement with those simulated by the FDTD directly.

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II. Analysis

The transient characteristic impedance, $z_0(t)$, and delay function, $h(t)$, are defined in the form of convolution as follow, respectively

$$v_1^+(t) = z_0(t) * i_1^+(t) = \int_0^t i_1^+(t-\lambda) z_0(\lambda)\, d\lambda \quad (1)$$

$$v_2^+(t) = h(t) * v_1^+(t) = \int_0^t v_1^+(t-\lambda) h(\lambda)\, d\lambda \quad (2)$$

where $v_1^+(t), v_2^+(t)$, and $i_1^+(t)$ are travelling wave voltages at observing plane 1 and 2 and current at plane 1 of a microstrip line, respectively. It is assumed that $v(t) = 0, i(t) = 0$ when $t < 0$.

Solving the convolution-type integral equations (1) and (2) to get z_0 and $h(t)$ with $v(t)$ and $i(t)$ is called deconvolution. Since $v(t)$ and $i(t)$ can easily be obtained numerically by FDTD, it is convenient to solve (1) and (2) numerically. Without loss of generality, we only consider (1). Assume

$$z_0(t) = \alpha\delta(t) + \tilde{z}(t) \quad (3)$$

which is the form of most applications, where $\delta(t)$ is the Dirac delta function, $\tilde{z}(t)$ is a nonsingular function, and α is a constant. Dividing the interval into N equally space regions, the width of every region, Δt, is small enough so that $i_1^+(t)$ varies very slowly in Δt, and (1) can be approximated as the well-known convolution summation

$$v_1^+(j\Delta t) = \sum_{k=1}^{j} i_1^+((j-k+1)\Delta t) z_a(k\Delta t) \quad (4)$$

where $j = 1, 2, \cdots, N$

$$z_a(k\Delta t) = \begin{cases} \alpha + \int_0^{\Delta t} z_0(\lambda)\, d\lambda & k = 1 \\ \int_{(k-1)\Delta t}^{k\Delta t} z_0(\lambda)\, d\lambda & k > 1. \end{cases} \quad (5)$$

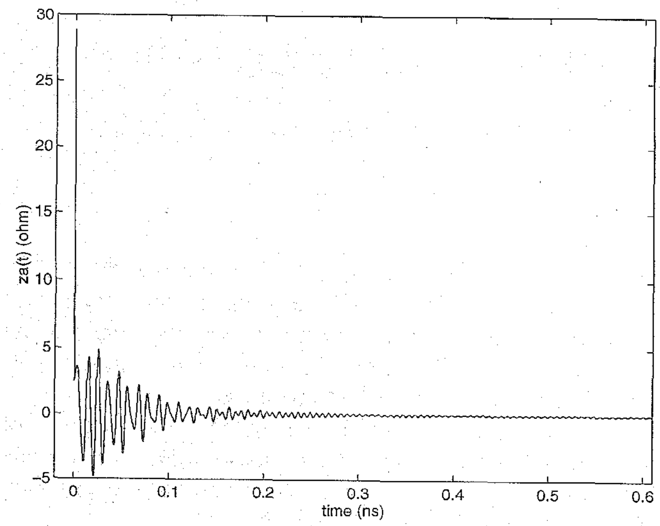


Fig. 1. Integral transient characteristic impedance.

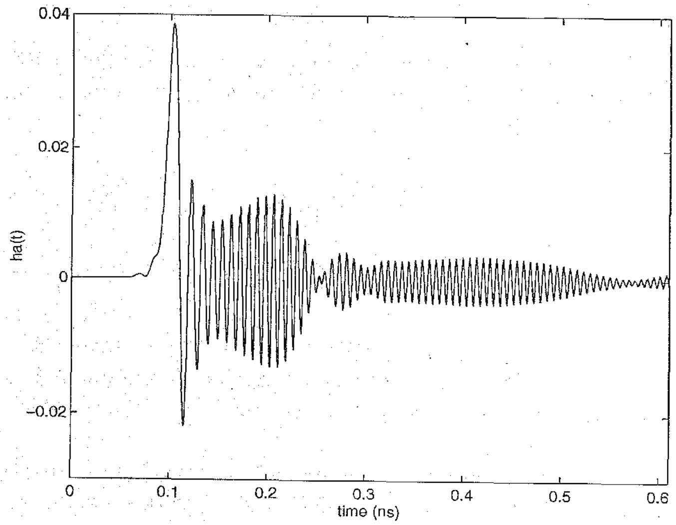


Fig. 2. Integral transient delay function.

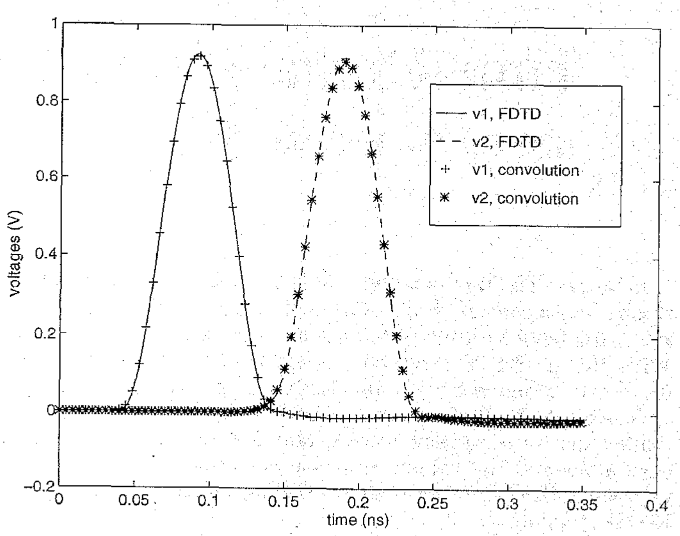
From (4), z_a may be written

$$z_a(j\Delta t) = \frac{v_1^+(j\Delta t) - \sum_{k=1}^{j-1} i_1^+((j-k+1)\Delta t) z_a(k\Delta t)}{i_1^+(\Delta t)}. \tag{6}$$

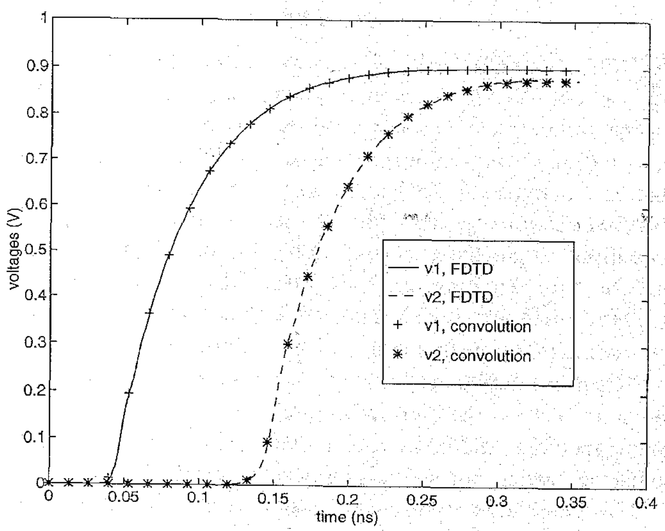
We can see from (4) that it is unnecessary to find $z_0(t)$. In the discrete system, we only need to find $z_a(j\Delta t)$, which is called integral transient characteristic impedance. It must be noted that the deconvolution operation of (6) is very sensitive to errors of $v_1^+(t)$ and $i_1^+(t)$ [5], [6]. If $v_1^+(t)$ and $i_1^+(t)$ are fairly small in magnitude (on the same order as the error), solving (6) for z_a by back substitution techniques usually leads to solutions that diverge. In a similar manner, we can obtain the integral transient delay function, $h_a(j\Delta t)$.

III. Numerical Results and Application

The parameters of the microstrip line used for our computations are thickness of the substrate ($H = 0.794$ mm), width of the metal strip ($W = 2.413$ mm), dielectric constant of the substrate ($\epsilon_r = 2.2$), and thickness of the metal strip (zero). The FDTD method is used to simulate the response voltages and currents. The details of FDTD algorithm used to simulate microstrip response have been discussed in numerous articles [7], [8] and will not be discussed here. The space steps are $\Delta x = 0.4046$ mm, $\Delta y = 0.4233$ mm, and $\Delta z = 0.2650$ mm. The line length between observing planes is $\ell = 50\Delta y$. The total mesh dimensions are $30 \times 200 \times 10$ in the $\hat{x}, \hat{y}$, and $\hat{z}$ directions, respectively. The first-order Mur's absorbing boundary condition is adopted to truncate the computing region. A linear excitation $e_\ell(t) = 0.001t$ is used at the front surface, uniform under the strip with only vertical electric field component E_z. The observing plane 1 is $50\Delta y$ distance from the excitation plane to allow only the dominant mode to propagate in the microstrip line. The distance between observing plane 2 and end absorbing boundary is $100\Delta y$ to reduce the effect of ABC. The time step is $\Delta t = 0.441$ ps. The simulation is performed for 2000 time steps. The computation time is approximately 20 min on a Sunstation model Ultra1.



(a)



(b)

Fig. 3. Comparison of response voltages by FDTD and by convolution summation (a) $e_c(t)$ excitation (b) $e_e(t)$ excitation.

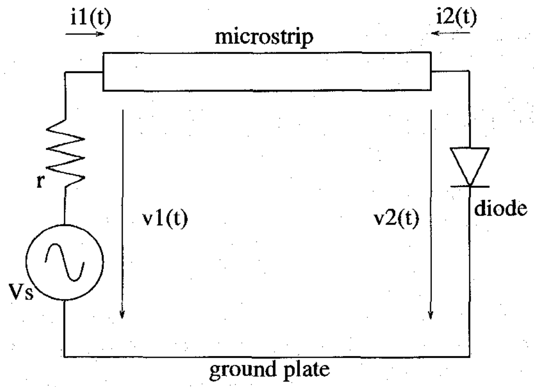


Fig. 4. Generic geometry used for simulation of response of diode terminating microstrip line.

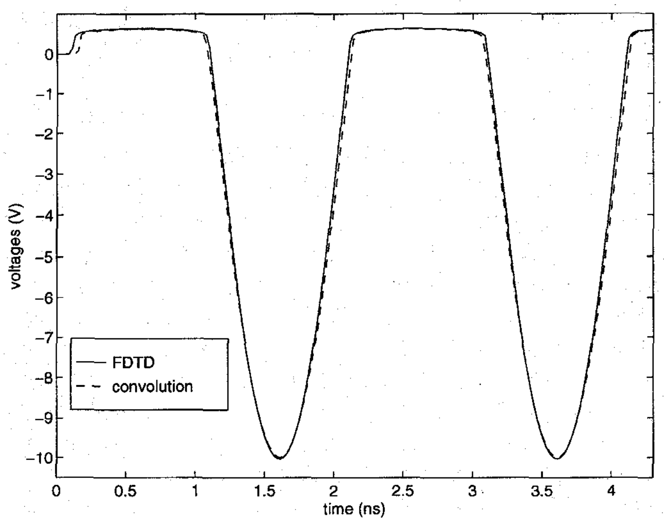


Fig. 5. Response voltage on diode terminating microstrip line.

By means of the linear response voltages and currents at observing planes, we obtain the integral transient characteristic impedance and delay function from (6) in a few seconds, as shown in Fig. 1 and Fig. 2, respectively. The initial response values $v(\Delta t)$ and $i(\Delta t)$ are appropriately chosen to be greater than 10^{-3} to avoid the numerical instability of deconvolution.

In order to examine our results, we also simulate the responses of the following excitations by the FDTD

$$e_e(t) = 1 - \exp(-bt) \tag{7}$$

$$e_c(t) = \begin{cases} 1 - \cos(2\pi f_c t) & 0 \le t \le \dfrac{1}{f_c} \\ 0 & t > \dfrac{1}{f_c}. \end{cases} \tag{8}$$

In our computation, we choose $b = 0.02$/ps, $f_c = 22.67$ GHz. Meanwhile, we compute the response voltages by convolution summation (4), in which $i_1^+(t)$ comes from FDTD simulation. The comparisons of two methods are shown in Fig. 3, which have good agreement.

As an example applying the time-domain model, we simulate the time-domain response of a diode terminating a microstrip line with a matched source as shown in Fig. 4. The relation of $v_2(t)$ and $i_2(t)$ is

$$\begin{aligned}(rh^-(t) + z_0(t) * h^+(t)) * v_2(t) - (rh^+(t)\\ + z_0(t) * h^-(t)) * z_0(t) * i_2(t) = 2h(t) * v_s(t) * z_0(t)\end{aligned} \tag{9}$$

where $h^{\pm}(t) = \delta(t) \pm h(t) * h(t)$. The current through the diode is expressed by

$$i_2(t) = -I_o[\exp(qv_2(t)/kT) - 1]. \tag{10}$$

In our computation, $I_o = 1\ \mu\text{A}, kT/q = 2 \times 26$ mV. The matched source is chosen as $r = 50\ \Omega$, and

$$v_s = 10\sin(2\pi f_o t) \tag{11}$$

where $f_o = 500$ MHz.

Substituting (10) and (11) into (9) and using convolution summation, we obtain the voltage on the diode in a few minutes, by means of numerical methods, solving nonlinear equation such as Newton's method. In order to compare, we also carry out FDTD simulation by the method in [3], which takes about 45 min. As shown in Fig. 5, these results agree well.

IV. Conclusion

The integral transient characteristic impedance and delay function of a microstrip line are obtained by means of the deconvolution of response voltages and currents simulated by the FDTD. The results have shown that this method is effective for the time-domain model synthesis of a microstrip line. With the model, the responses of any excitation for microstrip line loaded with any components or devices can be easily and rapidly obtained. Extending the method to various VLSI interconnects is being explored.

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